

## Ambipolar diffusion in complex plasma

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A self-consistent model of the ambipolar diffusion of electrons and ions in complex (dusty) plasmas accounting for the local electric fields, the dust grain charging process, and the interaction of the plasma particles with the dust grains and neutrals is presented. The dependence of the diffusion coefficient on the interaction of the electrons and ions with the dust grains as well as with the neutrals are investigated. It is shown that increase of the dust density leads to a reduction of the diffusion scale length, and this effect is enhanced at higher electron densities. The dependence of the diffusion scale length on the neutral gas pressure is found to be given by a power law, where the absolute value of the power exponent decreases with increase of the dust density. The electric field gradient and its effects are shown to be significant and should thus be taken into account in studies of complex plasmas with not very small dust densities. The possibility of observing localized coherent dissipative nonlinear dust ion-acoustic structures in an asymmetrically discharged double plasma is discussed.

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A phenomenon unique to weakly ionized plasmas is that of ambipolar diffusion [1]. It occurs when collisions of the charged plasma particles (electrons and ions) with each other are negligible in comparison to that with the neutral particles. The diffusion of the electrons and the ions are in this case highly correlated because of the appearance of a space-charge electric field [2].

Most naturally occurring weakly ionized plasmas are complex since besides the electrons, ions, and neutral atoms and/or molecules, they also contain massive electrically charged (dust) grains or microparticles in solid or liquid phase [3–9]. Complex or dusty plasmas are ubiquitous in space [10]. They also appear frequently in the laboratory as well as in the ionosphere [11,12]. Under different environmental conditions, these plasmas can behave differently. For example, it has been shown [13] that in the Earth's mesopause region, which contains charged dust grains [14], ambipolar diffusion of the ions and electrons makes the plasma there quite different from that in the laboratory.

An important property of a complex plasmas is the dust-grain charging process. Usually a dust grain is negatively charged because of the much higher (with respect to the ion) electron mobility, since its charge is determined by the fluxes (grain currents) of electrons and ions flowing onto its surface. Since the grain currents strongly depend on the local plasma parameters, the grain charge can be both space and time dependent. It also strongly depends on the recombination of the electrons and ions that hit the grain surface, as well as the momentum loss by the plasma particles from (including Coulomb) collisions with the grain. These processes are the most important relaxation mechanisms in a complex plasma [15]. Since diffusion, especially ambipolar diffusion, is often responsible for the formation of stationary or quasistationary plasma states [1,16], processes associated with dust charging, which strongly affect the diffusion of the

electrons and ions, can significantly affect a plasma containing dust grains.

Diffusion of dust grains was studied experimentally for quasi-two-dimensional complex plasmas [17] as well as under microgravity conditions on the MIR space station [16]. Earlier works on the diffusion of electrons and ions in complex plasmas [18,19] give theoretical results which roughly agree with some experimental data. For example, the presence of dusts in a plasma can lead to an appreciable reduction in the diffusion scale length [18].

A self-consistent model of ambipolar diffusion in complex plasmas still does not exist. Earlier studies [18,19] are based on the plasma quasineutrality assumption. An empirical momentum-transfer rate, which determines the diffusion coefficient, of the plasma particles, was also used. It includes only the nonelastic (impact) collisions of the electrons and ions with the dust grains, but not the elastic Coulomb collisions. The purpose of the present paper is to develop a more self-consistent model for the ambipolar diffusion of electrons and ions in a complex plasma by taking into account the dust charging process, the interaction of the plasma particles with the dust grains and neutrals, the resulting self-consistent electric field, etc. The model is then used to consider propagation of localized nonlinear structures such as dust ion-acoustic wave (DIAW) solitons and shocks in an inhomogeneous complex plasma.

We are interested in the ambipolar diffusion of the plasma electrons and ions, which are much lighter than the dust grains. Accordingly, we assume that the dust grain distribution remains stationary and its density profile flat. Furthermore, since we are interested in ambipolar diffusion, we shall assume that electron-ion, electron-electron, and ion-ion collisions are negligible.

The diffusion coefficients of the electrons ( $D_e$ ) and ions ( $D_i$ ) satisfy the Einstein relations [1,2]

$$D_e = T_e \mu_e / e \quad \text{and} \quad D_i = T_i \mu_i / e, \quad (1)$$

where  $T_{e,i}$  and  $\mu_{e,i}$  are the temperatures and mobilities of the electrons and ions, respectively, and  $-e$  is the electron charge. The electron and ion mobilities relate the drift velocities of the electrons and ions to the electric field. To determine these relations we shall make use of the fluid momentum-transfer equations for complex plasmas [20–24]. We also take into account the interaction of the plasma particles with the microparticles and neutrals, and look for balance between these processes, including that of acceleration of the electrons and ions in the self-consistent electric field  $\mathbf{E}$ . Accordingly, we obtain

$$\mu_e = e/m_e \nu_e^{\text{eff}} \quad \text{and} \quad \mu_i = e/m_i \nu_i^{\text{eff}}, \quad (2)$$

where  $m_e$  and  $m_i$  are the electron and ion masses,  $\nu_e^{\text{eff}} = \nu_{en} + \nu_{ed}$ ,  $\nu_i^{\text{eff}} = \nu_{in} + \nu_{id}$ ,  $\nu_{en}$  and  $\nu_{in}$  are the electron-neutral and ion-neutral collision frequencies, respectively, and  $\nu_{ed}$  and  $\nu_{id}$  are the momentum-transfer rates due to electron and ion collisions, respectively, with the dust grains.

We shall assume that the electron drift velocity  $\mathbf{u}_e$  is much smaller than its thermal velocity  $v_{Te} = \sqrt{T_e/m_e}$ , and the ion drift velocity  $\mathbf{u}_i$  can be arbitrary. The momentum-transfer rates due to the interaction of the plasma particles with the dust grains are then [23,24]

$$\nu_{ed} = (2\sqrt{2\pi/3})a^2 v_{Ti} n_d (n_i/n_e) (1+z\tau) \times [4+2z+z^2 \exp(z)\Lambda_{ed}], \quad (3)$$

$$\nu_{id}^{\text{coll}} = \sqrt{2\pi} a^2 v_{Ti} n_d \tilde{u}^{-2} \{ \sqrt{\pi/2} \operatorname{erf}(\tilde{u}/\sqrt{2}) \tilde{u} [1+\tilde{u}^2 + (1-\tilde{u}^2) \times (1+2\tau z)] + (1+2\tau z + \tilde{u}^2) \exp(-\tilde{u}^2/2) \}, \quad (4)$$

$$\nu_{id}^{\text{orb}} = \sqrt{2\pi} a^2 v_{Ti} n_d (2\tau z)^2 \Lambda_{id}(\tilde{u}) \tilde{u}^{-3} \times [ \sqrt{\pi/2} \operatorname{erf}(\tilde{u}/\sqrt{2}) - \tilde{u} \exp(-\tilde{u}^2/2) ], \quad (5)$$

where  $\nu_{id} = \nu_{id}^{\text{coll}} + \nu_{id}^{\text{orb}}$ ,  $\nu_{id}^{\text{coll}}$  is the ion momentum-transfer frequency due to the collection (absorption) of the ions by the dust grain,  $\nu_{id}^{\text{orb}}$  is the ion momentum-transfer rate of elastic ion Coulomb scattering in the dust-grain field,  $a$  is the dust radius,  $v_{Ti} = \sqrt{T_i/m_i}$  is the ion thermal velocity,  $n_d$ ,  $n_e$ , and  $n_i$  are the dust, electron, and ion densities, respectively,  $z = Z_d e^2 / a T_e$  is the dust surface potential in units of  $T_e/e$ ,  $q_d = -Z_d e$  is the dust charge,  $\tau = T_e/T_i$  is the electron-to-ion temperature ratio,  $\tilde{u} = |\mathbf{u}_i|/v_{Ti}$  is the normalized ion drift velocity,

$$\Lambda_{ed} = \int_0^\infty e^{-x} \ln \left( 1 + 4 \frac{\lambda_D^2 x^2}{a^2 z^2} \right) dx - 2 \int_z^\infty e^{-x} \ln \left( \frac{2x}{z} - 1 \right) dx$$

is the Coulomb logarithm for electron-dust collisions [24],  $\Lambda_{id}(\tilde{u}) \sim \ln\{(1+\beta)/[a/\tilde{\lambda}_D(\tilde{u})+\beta]\}$  is the Coulomb logarithm for ion-dust collisions [25],  $\beta(\tilde{u}) = z\tau[a/\tilde{\lambda}_D(\tilde{u})](1+\tilde{u}^2)^{-1}$ ,  $\tilde{\lambda}_D(\tilde{u})$  is the effective screening length defined by  $\tilde{\lambda}_D^{-2} = \lambda_{Di}^{-2}(1+\tilde{u}^2)^{-1} + \lambda_{De}^{-2}$ ,  $\lambda_D = \tilde{\lambda}_D(\tilde{u})|_{\tilde{u}=0}$ , and  $\lambda_{De,i} = \sqrt{T_{e,i}/4\pi e^2 n_{e,i}}$  is the electron or ion Debye length.

The value of  $\Lambda_{ed}$  can be approximated (within  $\sim 30\%$  accuracy) by the simplified relation  $\Lambda_{ed} \approx 2 \ln(\lambda_D/a)$  [24] which is contained in the electron-dust momentum transfer

rates used in Refs. [20–23]. The expression for  $\Lambda_{id}(\tilde{u})$  differs from usual because of the much larger ion-dust interaction range (the Coulomb radius) associated with the elastic Coulomb scattering. It reduces to the standard expression in the limit  $\beta \ll 1$  [26].

The electron and ion diffusion fluxes are given by [1]

$$\Gamma_e \equiv n_e \mathbf{u}_e = -D_e \nabla n_e - \mu_e n_e \mathbf{E}, \quad (6)$$

$$\Gamma_i \equiv n_i \mathbf{u}_i = -D_i \nabla n_i + \mu_i n_i \mathbf{E}. \quad (7)$$

For ambipolar diffusion, the electron and ion fluxes are equal, or

$$\Gamma_e = \Gamma_i = \Gamma. \quad (8)$$

In the steady state, the electron continuity equation is

$$\nabla \cdot \Gamma = -\nu_r n_e + \nu_I n_e - \beta_{ei} n_i n_e - \nu_{\text{loss}} n_e, \quad (9)$$

where

$$\nu_r = 2\sqrt{2\pi} a^2 v_{Ti} n_d (n_i/n_e) (1+z\tau), \quad (10)$$

is the rate of electron recombination on the dust grain,  $\nu_I$  is the ionization frequency,  $\beta_{ei}$  is the rate of recombination in the plasma bulk, and  $\nu_{\text{loss}}$  is a generalized loss rate. Bulk recombination are usually negligible for low-pressure discharges, so that Eq. (10) can be rewritten in the form

$$\nabla \cdot \Gamma = -\nu^* n_e, \quad (11)$$

where  $\nu^* = \nu_r - \nu_I + \nu_{\text{loss}}$ . In the following we shall concentrate on Eq. (11).

Ambipolar diffusion is also affected by the dust charging process. Under most experimental conditions the dust-charge variation is mainly due to the variation of the local plasma potential. In this case dust charging can be roughly described by the orbit-motion-limited (OML) theory [21,27]. In the steady state, the dust charge is then determined by a balance of the local electron ( $J_e$ ) and ion ( $J_i$ ) charging fluxes hitting the dust surface, or

$$J_e - J_i = 0, \quad (12)$$

where

$$J_e = 2\sqrt{2\pi} a^2 v_{Te} n_e \exp(-z) \quad (13)$$

and

$$J_i = \sqrt{2\pi} a^2 v_{Ti} n_i \tilde{u}^{-1} \{ \tilde{u} \exp(-\tilde{u}^2/2) + \sqrt{\pi/2} \operatorname{erf}(\tilde{u}/\sqrt{2}) \times [1 + \tau z + \tilde{u}^2] \}. \quad (14)$$

We note that to refine on the description of dust charging one has often to take into account the effect of ion-neutral charge-exchange collisions in the vicinity of dust grain that can lead to a substantial current to its surface [28–31]. However, in the present paper we consider as a specific example a plasma in an asymmetrically discharged double-plasma device [32], where the typical pressure is only several tenths Pa. For such low pressures and micrometer-size dust particles, this effect is not significant.

Equations (1)–(14) are closed by the Poisson equation,

$$\nabla \cdot \mathbf{E} = 4\pi e(n_i - n_e - Z_d n_d). \quad (15)$$

The steady-state diffusion profiles are a solution of Eqs. (6)–(8), (11), (12), and (15). We shall solve this set of equations numerically. It is convenient to introduce the dimensionless variables and parameters  $\bar{x} = x/\lambda_{De}$ ,  $\bar{E} = e\lambda_{De}E/T_e$ ,  $\bar{u}_{e,i} = u_{e,i}/c_s$ ,  $\bar{n}_{e,i,d} = n_{e,i,d}/n_{e0}$ ,  $\bar{\Gamma} = \Gamma\lambda_{De}v_i^{\text{eff}}/n_{e0}c_s^2$ ,  $f = \ln(\bar{n}_e)$ ,  $g = \ln(\bar{n}_i)$ ,  $\xi = (m_e/m_i)(v_e^{\text{eff}}/v_i^{\text{eff}})$ ,  $\eta = \lambda_{De}^2/L_D^2$ , where  $n_{e0}$  is a reference electron density,  $c_s = \sqrt{T_e/m_i}$  and  $L_D = c_s/\sqrt{\nu^* v_i^{\text{eff}}}$  are the ion acoustic speed and the ambipolar diffusion length for a simple electron-ion plasma [18], respectively. Note that, following tradition and for ease of comparison with simple plasmas, the normalization parameters such as  $\lambda_{De}$ ,  $v_i^{\text{eff}}$ , etc., are that associated with the latter plasma and do not directly reflect the physical properties of the complex plasma under consideration. Accordingly, Eqs. (6)–(8), (11), and (15) become

$$\frac{\partial f}{\partial \bar{x}} + \bar{E} = -\bar{\Gamma}\xi \exp(-f), \quad (16)$$

$$\frac{1}{\tau} \frac{\partial g}{\partial \bar{x}} - \bar{E} = -\bar{\Gamma} \exp(-g), \quad (17)$$

$$\frac{\partial \bar{\Gamma}}{\partial \bar{x}} = -\eta \exp(f), \quad (18)$$

$$\frac{\partial \bar{E}}{\partial \bar{x}} = \exp(g) - \exp(f) - Z_d \bar{n}_d. \quad (19)$$

We shall assume  $\xi \ll 1$ , which is valid for a wide range of complex-plasma parameters. Adding Eqs. (16) and (17), differentiating their sum and Eq. (16) by  $\bar{x}$ , and using the resulting equations and Eq. (19), we can eliminate the electric field  $\bar{E}$ . Neglecting the term  $(1/\tau)\partial_{\bar{x}}^2 g$  in comparison with  $\partial_{\bar{x}}^2 f$  (valid for  $\tau \gg 1$  or  $T_i \ll T_e$ ), we obtain

$$\frac{\partial g}{\partial \bar{x}} = \frac{1}{\bar{\Gamma}} [\exp(f+g) - \eta \exp(f) - \exp(2g) + Z_d \bar{n}_d], \quad (20)$$

$$\begin{aligned} \frac{\partial f}{\partial \bar{x}} = & -\frac{1}{\tau \bar{\Gamma}} [\exp(f+g) - \eta \exp(f) - \exp(2g) + Z_d \bar{n}_d] \\ & - \bar{\Gamma} \exp(-g), \end{aligned} \quad (21)$$

which together with Eq. (12) provide a self-consistent description of the diffusion process in a complex plasma with  $T_i \ll T_e$ . This set of equations is solved numerically by a fourth-order Runge-Kutta method [33]. The boundary conditions are the plasma neutrality condition and  $n_e = 2n_{e0}$  at  $\bar{x} = 0$  and  $\partial_{\bar{x}} n_e \rightarrow 0$  for  $\bar{x} \rightarrow \infty$ .

We now apply our diffusion model to consider the properties of an inhomogeneous complex plasma, in particular that in an asymmetrically discharged double-plasma device [32]. We shall focus on the effect of the dust grains on the diffusion scale length and the electric field that appears self-consistently because of the diffusion process. For concrete-

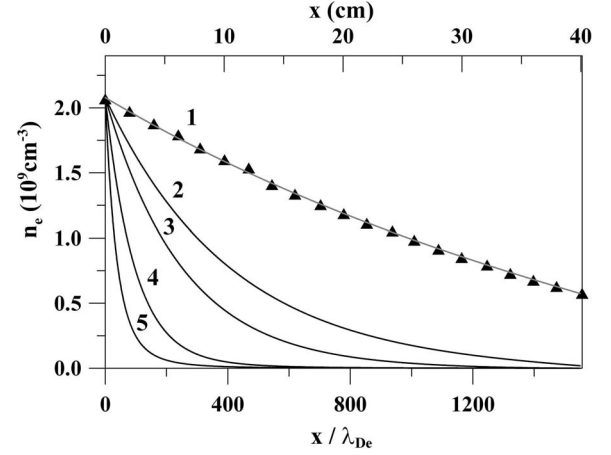


FIG. 1. The diffusion profiles of the electron density for  $n_d=0$  (curve 1),  $n_d=10^2 \text{ cm}^{-3}$  (curve 2),  $n_d=10^3 \text{ cm}^{-3}$  (curve 3),  $n_d=10^4 \text{ cm}^{-3}$  (curve 4), and  $n_d=10^5 \text{ cm}^{-3}$  (curve 5) in an argon plasma with argon gas pressure 0.18 Pa,  $n_{e0}=10^9 \text{ cm}^{-3}$ ,  $T_e=1.2 \text{ eV}$ , and  $T_i=0.03 \text{ eV}$ . The dust grain radius is  $a=4.4 \mu\text{m}$ . The triangles are from the experiments of Ma *et al.* [32] for  $n_d=0$ .

ness, we consider an argon plasma with the electron and ion temperatures  $T_e=1.2 \text{ eV}$  and  $T_i=0.03 \text{ eV}$ , respectively. The grain radius is taken to be  $a=4.4 \mu\text{m}$ .

Figure 1 shows the diffusion generated profiles of the electron density for different dust densities. The argon gas pressure is 0.18 Pa, and the background electron density is  $n_{e0}=10^9 \text{ cm}^{-3}$ . The triangles are the experimental data of Ma *et al.* [32] for  $n_d=0$ . There is a rather good agreement between the theoretical curve 1 (for  $n_d=0$ ) and the experimental data.

Figure 2 shows the profiles of the electric field associated with the diffusion process for the same parameters as in Fig. 1. The profiles 1–4 correspond to the electron density profiles 2–5 in Fig. 1, respectively. We note that for not small dust densities, say,  $n_d \gg 10^3 \text{ cm}^{-3}$ , the electric field gradient

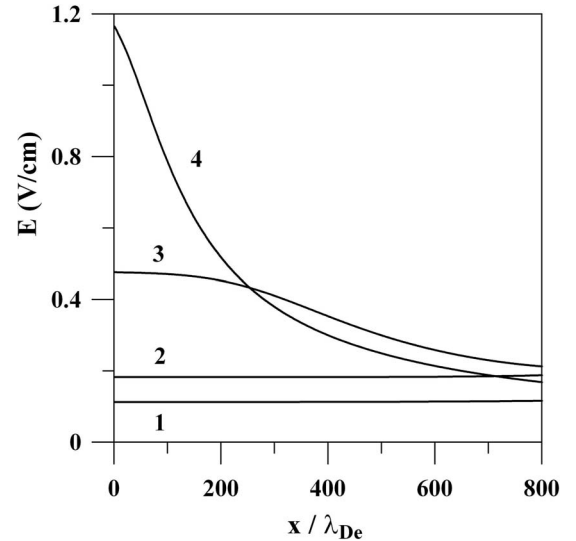


FIG. 2. The electric field profiles for  $n_d=10^2 \text{ cm}^{-3}$  (curve 1),  $n_d=10^3 \text{ cm}^{-3}$  (curve 2),  $n_d=10^4 \text{ cm}^{-3}$  (curve 3), and  $n_d=10^5 \text{ cm}^{-3}$  (curve 4). The other parameters are the same as in Fig. 1.

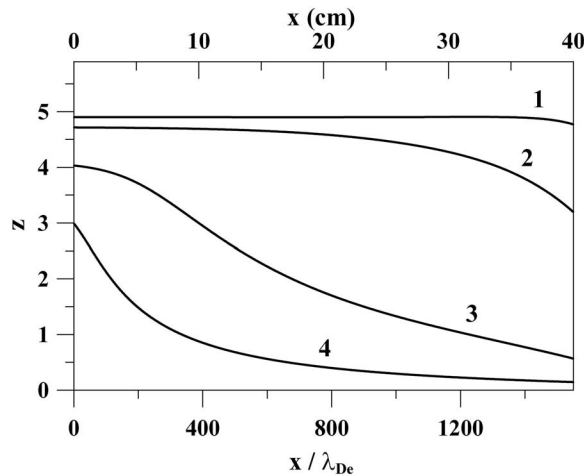


FIG. 3. The ambipolar diffusion dust grain charge distributions for  $n_d=10^2 \text{ cm}^{-3}$  (curve 1),  $n_d=10^3 \text{ cm}^{-3}$  (curve 2),  $n_d=10^4 \text{ cm}^{-3}$  (curve 3),  $n_d=10^5 \text{ cm}^{-3}$  (curve 4). The other parameters are the same as in Fig. 1.

is not small and should thus be taken into account explicitly in related theoretical studies.

Figure 3 gives the dust charge distribution in the presence of ambipolar diffusion. Analysis of Eq. (12) shows that for the plasma considered (argon with  $T_e=1.2 \text{ eV}$ ,  $T_i=0.03 \text{ eV}$ , and  $a=4.4 \mu\text{m}$ ) the maximum dust charge  $z_{\text{max}}=4.92$  is attained for  $|\mathbf{u}_i|=4.45v_{Ti}$  and  $n_{i0}=n_{e0}$ . That is, the maximum dust grain charge can be attained only if the plasma quasineutrality condition is violated because the plasma contains charged dusts in addition to the electrons and ions. The value  $z_{\text{max}}$  is shown (see curve 1 in Fig. 3) to be attained for the dust density  $n_d=10^2 \text{ cm}^{-3}$ , thus confirming the violation of the plasma quasineutrality condition and the importance of the electric field associated with ambipolar diffusion.

The dependence of the ambipolar diffusion length  $\lambda_a$ , i.e., the distance at which an  $e$ -fold decrease in the electron density occurs, on the dust density for different  $n_{e0}$  is shown in Fig. 4. We see that increase of the dust density is accompanied by reduction of the diffusion scale length, in good agreement with the experimental results [18,34]. The effect of this reduction is stronger for larger electron densities. This result can be attributed to an increase (due to increase of the dust density) of the contribution of the electron and ion collisions with the dust grains in comparison to that with the neutrals.

Figure 5 shows the dependence of the ratio  $\lambda_a/\lambda_{De}$  on the pressure  $P$  of the argon gas for  $n_{e0}=10^9 \text{ cm}^{-3}$  and different dust densities. All the cases satisfy the power law  $\lambda_a/\lambda_{De} \propto P^{-\alpha}$ , namely,  $\alpha=-0.93$  for  $n_d=10^2 \text{ cm}^{-3}$  (curve 1),  $\alpha=-0.71$  for  $n_d=10^3 \text{ cm}^{-3}$  (curve 2), and  $\alpha=-0.54$  for  $n_d=10^4 \text{ cm}^{-3}$  (curve 3). The magnitude of the power exponent increases with decrease of the dust density. In particular, we have  $\alpha=-1$  for  $n_d=0$ , as also found experimentally by Ma *et al.* [32].

In the above we have presented and discussed a self-consistent model of the ambipolar diffusion for inhomogeneous complex plasmas. We shall now apply the results to obtain the conditions for the existence of DIAW solitons and

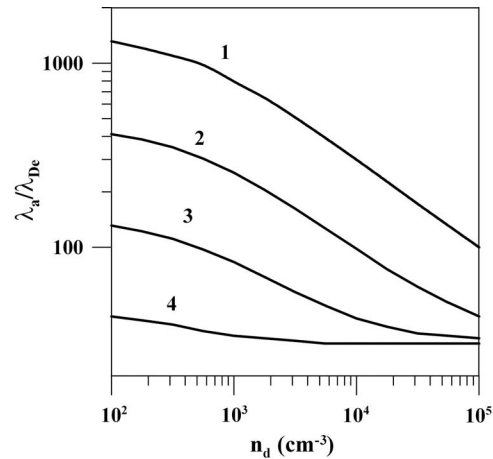


FIG. 4. The ambipolar diffusion length versus the dust density for  $n_{e0}=10^{10} \text{ cm}^{-3}$  (curve 1),  $n_{e0}=10^9 \text{ cm}^{-3}$  (curve 2),  $n_{e0}=10^8 \text{ cm}^{-3}$  (curve 3),  $n_{e0}=10^7 \text{ cm}^{-3}$  (curve 4). The other parameters are the same as in Fig. 1.

shocks in such a plasma. Double plasmas have often been used for investigating localized nonlinear structures such as the dust ion-acoustic wave (DIAW) shocks [35] and solitons [36] in homogeneous plasmas. In most applications, however, the plasma is inhomogeneous. Clearly, a localized structure can be excited in an inhomogeneous plasma only if the diffusion scale length of the plasma is much smaller than its characteristic width.

The width  $\Delta\xi_{\text{sol}}$  of a DIAW soliton is mainly determined by the electron Debye length (see, e.g., Ref. [37]). For the parameters here we have  $\Delta\xi_{\text{sol}} \sim 10\lambda_{De}$ . From the data in Fig. 4, we can conclude that for the asymmetrically discharged double-plasma device [32], namely,  $P=0.18 \text{ Pa}$ ,  $T_e=1.2 \text{ eV}$ ,  $T_i=0.03 \text{ eV}$ , and  $a=4.4 \mu\text{m}$ , DIAW solitons may be observed only for relatively large electron densities, namely,  $n_{e0} > 10^8 \text{ cm}^{-3}$  and relatively low dust densities, namely  $n_d \ll 10^4 \text{ cm}^{-3}$ .

For the parameters here, one of the most relevant DIAW shocks is that related to anomalous dissipation that originates

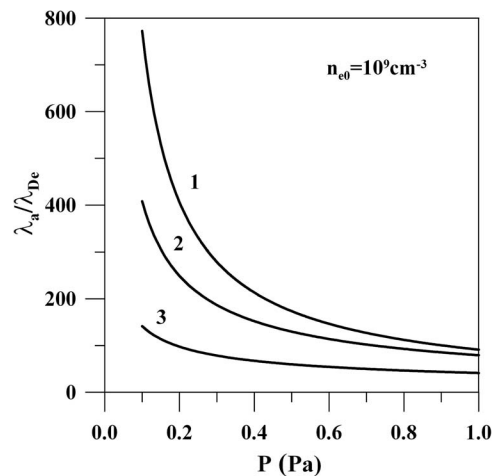


FIG. 5. The ratio  $\lambda_a/\lambda_{De}$  versus the argon gas pressure  $P$  for  $n_d=10^2 \text{ cm}^{-3}$  (curve 1),  $n_d=10^3 \text{ cm}^{-3}$  (curve 2),  $n_d=10^4 \text{ cm}^{-3}$  (curve 3). The other parameters are the same as in Fig. 1.

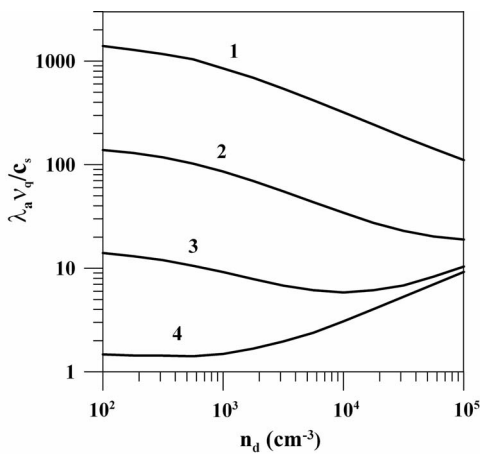


FIG. 6. The ratio  $\lambda_a v_q / c_s$  versus the dust density for  $n_{e0} = 10^{10} \text{ cm}^{-3}$  (curve 1),  $n_{e0} = 10^9 \text{ cm}^{-3}$  (curve 2),  $n_{e0} = 10^8 \text{ cm}^{-3}$  (curve 3),  $n_{e0} = 10^7 \text{ cm}^{-3}$  (curve 4). The other parameters are the same as in Fig. 1.

from the dust charging process [38,39]. This shock is especially interesting since it involves physical effects that exist only in complex plasmas. The width of the shock front  $\Delta \xi_{\text{sh}}$  can be estimated by  $\Delta \xi_{\text{sh}} \sim M c_s / v_q$  [38,39], where  $M c_s$  is the speed of the DIAW shock front,  $M$  is the Mach number,  $v_q = \omega_{pi}^2 a (1+z+T_i/T_e) / \sqrt{2\pi v_{Ti}}$  is the grain charging rate, and  $\omega_{pi} = \sqrt{4\pi n_i e^2 / m_i}$  is the ion plasma frequency. The Mach number  $M$  for DIAW shocks of not too large amplitude as observed in a homogeneous double plasma is of the order of unity [35]. Thus the necessary condition for these shocks to be excited in inhomogeneous complex plasmas is  $\lambda_a \gg c_s / v_q$ . Figure 6 shows the dependence of the ratio  $\lambda_a v_q / c_s$  on the dust density for different electron densities  $n_{e0}$ ,  $a = 4.4 \mu\text{m}$ , for the parameters of the inhomogeneous double plasma of interest here [32]. We see that DIAW shocks as-

sociated to anomalous dissipation can only appear if the electron density is large,  $n_{e0} \gg 10^8 \text{ cm}^{-3}$ .

In summary, we have introduced a self-consistent model for ambipolar diffusion of electrons and ions in complex plasmas accounting for the dust grain charging process, the interaction of the plasma particles with the dust grains and neutrals, the resulting self-consistent electric fields, etc., and studied its effect on the properties of an inhomogeneous plasma. The diffusion coefficient is strongly affected by the interaction of electrons and ions with the dust grains as well as the neutrals. It is found that the gradients of the self-consistent electric fields are not negligible and should be taken into account if the dust density is not small ( $n_d \gg 10^3 \text{ cm}^{-3}$ ). This effect implies the violation of the quasineutrality condition and is stronger at larger dust densities. The violation of the quasineutrality condition and the importance of the electric field associated with ambipolar diffusion are confirmed for dust densities as large as  $n_d = 10^2 \text{ cm}^{-3}$ . Increase of the dust density leads also to a reduction in the diffusion scale length and this effect is stronger at larger electron densities. It is found that the dependence of the diffusion scale length on the neutral gas pressure is well described by a power law  $\lambda_a \propto P^{-\alpha}$ , where the magnitude of the power decreases with increase of the dust density. We have also given the conditions for observation of DIAW solitons and shocks in an inhomogeneous plasma from an asymmetrically discharged double-plasma device [32]. It is found that DIAW solitons can be observed only for rather large ( $n_{e0} > 10^8 \text{ cm}^{-3}$ ) electron densities and rather low ( $n_d \ll 10^4 \text{ cm}^{-3}$ ) dust densities, while DIAW shocks can appear only at large ( $n_{e0} \gg 10^8 \text{ cm}^{-3}$ ) electron densities.

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